Title: State Lottery -- How Good Are Your Chances?

Brief Overview:

In this unit students will determine the probability of winning a state lottery (MD). They also will create a new set of rules to modify the lottery to fit two different situations. The students will use the graphing calculator to find the total number of possible outcomes using the probability menu. They will take part in a classroom simulation of the lottery. Finally, they will use oral or written communication to explain how their rule changes would affect the probability of winning.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

- **Number and Operation**
  The student will understand the relationship between numbers by comparing probabilities. The student will use computational tools and strategies fluently by selecting and using appropriate methods for computing (graphing calculator.)

- **Data Analysis, Statistics and Probability**
  The student will understand and apply basic notions of chance and probability by determining the probability of winning using different types of combinations.

Links to National Science Education Standards:

- **Science in Personal and Social Perspectives**
  The student will use technology appropriately to relate to a societal occurrence.

Grade/Level:

Grades 8-9

Duration/Length:

5-6 days

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Differentiating between a permutation and combination
- Calculating permutations and combinations
- Computing a simple probability

Student Outcomes:

Students will:

- work cooperatively in groups.
- use graphing calculator to determine possible outcomes of permutations and combinations.
- determine the probability of winning a state lottery (MD).
- determine the probability of winning and participate in a simulated lottery.
- analyze the results of the simulation to modify the rules for lotteries with different probabilities of winning.
- present the new rules to the class (using charts if desired).

Materials/Resources/Printed Materials:

- 1 state lottery ticket (Unit based on MD lottery of pick 6 from 48 numbers)
- 6 ping-pong balls (or 6 other items for simulation lottery)
- Graphing calculators (TI-80, -82, -73, -83)
- TI key stroke directions for combinations and permutations
- Transparencies #1 - 3
- Student Worksheets #1 - 3
- Teacher resource-day by day guide, lottery tickets,worksheet keys and rubric
- Peer assessment rubric sheet (Worksheet 4)
- Lottery prize (optional), e.g., bag of candy coins, paper money, extra credit points

Development/Procedures:

- **Day 1**:
  Teacher will announce a classroom lottery will be held. Students will review the difference between permutations and combinations and calculate them by completing a worksheet by hand. Teacher will instruct students on procedures to determine permutations and combinations on the graphing calculator. Students will then check their own worksheet using the graphing calculator.

- **Day 2**:
  Teacher will lead students in calculating the probability of two different types of lotteries using a graphing calculator. Students will determine the number of outcomes (and the probability) of winning the state lottery.

- **Day 3**:
  Students will participate in a simulation lottery after calculating total possible outcomes and probability of winning. Students will work in cooperative groups to brainstorm for making changes in rules to give first a greater chance and then a lesser chance of winning. Individually, students will complete the worksheet for homework.

- **Day 4:**
  Student volunteers will informally explain their answers to homework questions. Students will read the lottery assessment and rubrics, and teacher will answer any questions on requirements or rubrics. Students will decide which type of assessment to do and work individually on assessment (using homework). Teacher will decide the appropriate amount of time needed to complete the assessment.

- **Day 5:**
  Student will either turn in written prompt or present the rules for his modifications of the class lottery orally. Students will grade the oral presentations using the appropriate rubric.

Performance Assessment:

Students will create different variations on the classroom lottery that have greater and lesser probabilities of winning. Assessment will be evaluated based on clarity of explanation (communication) and on the quality of mathematical content. Oral presenters will receive additional feedback from peers.

Extension/Follow Up:

See Day-by-Day guide for possible discussion/performance questions.

Authors:

Debi Albright
Lindale/Brooklyn Park Middle School
Anne Arundel County, MD

WilliMae Brown
Lindale/Brooklyn Park Middle School
Anne Arundel County, MD

Robert Levering
Green Acres School
Montgomery County, MD

Cathi Sciantarelli
Lindale/Brooklyn Park Middle School
Anne Arundel County, MD

Day-by-Day Guide

Day 1

1. Students will review calculating permutations and combinations using Worksheet #1.

2. Teacher will instruct students how to use the TI calculator (use TI Calculator guide) to do these problems and have them check their own worksheet.

Day 2

Students will determine the possible outcomes for the state lottery (MD) through the following activities:

1. List all the possible combinations of picking out 2 numbered balls from a container of 4 numbered balls with replacement. (Tree diagram works well. See Transparency #1.)

1,2	2,1	3,1	4,1
1,3	2,3	3,2	4,2
1,4	2,4	3,4	4,3
1,1	2,2	3,3	4,4

The lottery is not based on replacement, so certain outcomes can be eliminated.
(1,1 2,2 3,3 4,4)
That leaves 12 possible outcomes.
The lottery is also not based on permutations, that is, order does not matter.
So eliminate the duplicates (1,2 and 2,1 are the same combination, so one of them is a duplicate.

1,2	1,3	1,4
2,3	2,4	3,4

That leaves you with 6 possible outcomes.
This can be solved as a combination ${}_4\mathbf{C}_2 = 6$
Discuss what the probability would be to win such a drawing. [1/6]

2. For practice have students figure out the combinations for picking 4 correct numbers out of 10 possible numbers. [210] (Use Transparency #2.)

3. Now compute the total possible outcomes of the state LOTTO game (picking 6 numbers out of 48) and the probability of having a winning ticket. Remember that you get 2 chances per ticket. (Transparency #3. If not in the state of Maryland, see what are the total numbers used in your state.)[12,271,512 outcomes]
[P(winning ticket) = 2/12,271,512 = 1/6135756]

Day 3

1. Time to hold your class LOTTO game.
   - Number 6 ping-pong balls with a 1 through 6.
   - Have students calculate the number of possible outcomes and the probability of winning when picking 4 out of 6 numbers. [20 possible outcomes]
     [P(Winning) = 1/20]
   - Hold the lottery.
   - Winners of the class lottery could be given some of those gold wrapped chocolate coins you can get at candy stores, homework pass, extra credit, etc.
   - Some discussion should take place on the number of winners in the class.
     Was it more or less than you expected?
     It will most likely be less bearing out that even at a 1/20 probability of winning, you still have a higher probability of losing: 19/20.

2. Give students Worksheet #2 and have students brainstorm new rules for making a lottery that would have a better probability of winning or a worse probability of winning.

3. Students will individually complete Worksheet #2 for homework.

Day 4

1. Student volunteers give sample solutions to homework problems without deep explanation of mathematical proof.

2. Teacher gives copy of lottery assessment and grading rubric(Worksheet #3) and answers any questions.

3. Teacher explains that final grade on assessment is based on both the clarity of explanation (communication) and mathematical content whether presentation is oral or written. As an added benefit to students, oral presentations can be critiqued by peers (Worksheet #4) if desired.

4. Students use rest of period to work on assessment of their choice.

Days 5/6

1. Students make oral presentations or turn in written prompt.

2. Oral presentations are evaluated by peers. (See Worksheet #4 for forms).

Extension

1. Discuss how you could be guaranteed to win the LOTTO.
   [If you could buy all the possible combinations.]

2. Research articles on the lottery using the Internet. (Good articles from the Richmond Times dated February 19 -26,1992 concerning the Australian syndicate's attempt to purchase all possible combinations in the Virginia lottery. Available for purchase.)

3. Calculate as a class how long it would take to print out all the possible combinations if one ticket can be printed each second.
   [6135756 seconds]
   [6135756/60 = 102262.6 minutes]
   [102262.6/60 = 1704.4 hours]
   [1704.4/24 = 71 days]
   So... is it possible to buy all the combinations?? Discuss.

4. Writing based on article
   How high would the jackpot have to be before you would purchase all the possible combinations?
   How much money would you actually receive (taking into account expenses and taxes of about 28%)?
   Based on what you have learned about probability, justify if it is to your advantage to play the lottery.

Using a TI Calculator to Calculate Permutations and Combinations

To solve permutation example: ${}_8P_4$

- Enter 8
- Press [MATH]
- Using cursor right, select PRB
- Using cursor down, select ${}_nP_r$. Press [ENTER]
- Enter 4
- Press [ENTER]

To solve combination example: ${}_8C_4$

- Enter 8
- Press [MATH]
- Using cursor right, select PRB
- Using cursor down, select ${}_nC_r$. Press [ENTER]
- Enter 4
- Press [ENTER]

Worksheet #1

Permutations & Combinations

Compute each permutation and combination.

1. ${}_8P_4$

2. ${}_{11}P_4$

3. ${}_9C_3$

4. ${}_5C_3$

Decide if the problem is an example of a permutation or combination. Then compute each.

5. How many teams of 4 horses would be made if there were 9 horses?

6. A lock has 30 numbers. How many different combinations for the lock are there if it uses 3 numbers?

7. Mike has nine baseball trophies to arrange on the shelf. How many different ways can they be arranged?

8. In math class there are 24 students. The teacher picks 4 students to do a presentation. How many different groups are possible?

9. In how many ways can ten people wait in line for concert tickets?

10. The teacher has 30 different books to choose from. You must read 4. How many different sets could you read?

Worksheet #1

Permutations & Combinations

Answer Key

Compute each permutation and combination.

1. ${}_8P_4 = 1680$

2. ${}_{11}P_4 = 7920$

3. ${}_9C_3 = 84$

4. ${}_5C_3 = 10$

Decide if the problem is an example of a permutation or combination. Then compute each.

5. How many teams of 4 horses would be made if there were 9 horses? C [126]

6. A lock has 30 numbers. How many different combinations for the lock are there if it uses 3 numbers?

P [24,360]

7. Mike has nine baseball trophies to arrange on the shelf. How many different ways can they be arranged?

P [362,880]

8. In math class there are 24 students. The teacher picks 4 students to do a presentation. How many different groups are possible?

C [10,626]

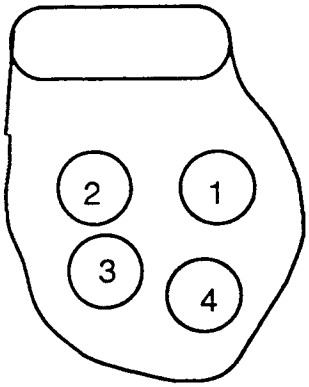
9. In how many ways can ten people wait in line for concert tickets?

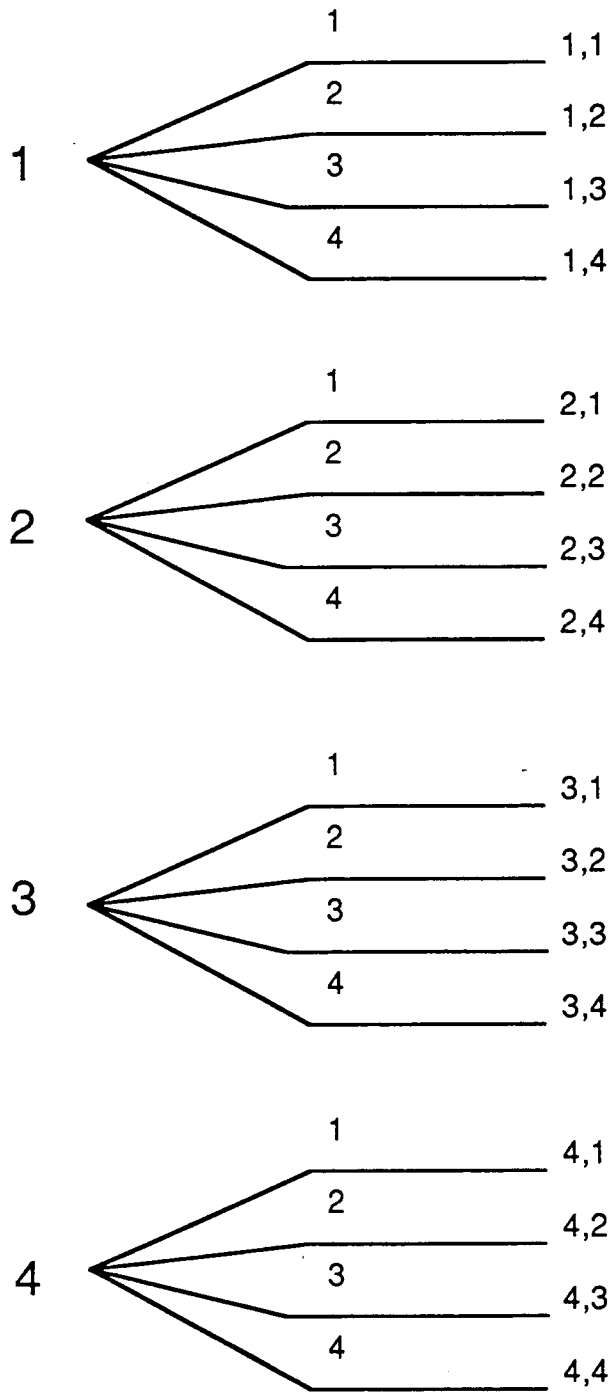
P [3,628,800]

10. The teacher has 30 different books to choose from. You must read 4. How many different sets could you read?

C [27,405]

MINI-LOTTO: 4 #'S PICK 2





“Pick 4 LOTTO”

Given: 10 numbers -- pick 4

How could you mathematically do this without using a tree diagram?

$$_{10}C_{4} =$$

$$_{10}C_{4} = \frac{10!}{(10-4)!\,4!}$$

$$_{10}C_{4} = \frac{10 * 9 * 8 * 7 * 6!}{(6!) * 4 * 3 * 2 * 1} = 210 \text{ outcomes}$$

What is the P(winning this LOTTO) ?

$$P(\text{winning}) = \frac{1}{210}$$

“Maryland LOTTO”

Given: 48 numbers -- pick 6

$$_{48}C_{6} =$$

Use the calculator to solve.

$$_{48}C_{6} = 12{,}271{,}512$$

What is the P(winning this LOTTO) ?

$$P(\text{winning}) = \frac{1}{12{,}271{,}512}$$

If you get 2 chances per ticket, what is the probability of having a winning ticket?

$$P(\text{winning ticket}) = \frac{2}{12{,}271{,}512} = \frac{1}{6{,}135{,}756}$$

$$$ $$$ $$$ **CLASS LOTTERY** NAME ________________ PICK 3 ___ ___ ___ $$$ $$$ $$$	$$$ $$$ $$$ **CLASS LOTTERY** NAME ________________ PICK 3 ___ ___ ___ $$$ $$$ $$$
$$$ $$$ $$$ **CLASS LOTTERY** NAME ________________ PICK 3 ___ ___ ___ $$$ $$$ $$$	$$$ $$$ $$$ **CLASS LOTTERY** NAME ________________ PICK 3 ___ ___ ___ $$$ $$$ $$$
$$$ $$$ $$$ **CLASS LOTTERY** NAME ________________ PICK 3 ___ ___ ___ $$$ $$$ $$$	$$$ $$$ $$$ **CLASS LOTTERY** NAME ________________ PICK 3 ___ ___ ___ $$$ $$$ $$$
$$$ $$$ $$$ **CLASS LOTTERY** NAME ________________ PICK 3 ___ ___ ___ $$$ $$$ $$$	$$$ $$$ $$$ **CLASS LOTTERY** NAME ________________ PICK 3 ___ ___ ___ $$$ $$$ $$$

HOMEWORK WORKSHEET

1. **How would you alter the rules and/or the structure of our class lottery game to make it easier for participants to win? List as many different ways as you can and explain each one using mathematical reasoning (that is, explain why your change would increase the mathematical chances of winning).**

2. **How could you alter the rules and/or structure of our class lottery game in order to decrease the chances of anyone winning? List as many different ways as you can and explain your reasoning mathematically.**

HOMEWORK WORKSHEET
Teacher's Resource

1. How would you alter the rules and/or the structure of our class lottery game to make it easier for participants to win? List as many different ways as you can and explain each one using mathematical reasoning (that is, why your change would increase the mathematical chances of winning).

SAMPLE ANSWER - You could increase the chances of winning by:

a) Giving each student two (or more) numbers instead of one. Since the number of possible numbers would remain the same, by giving a student two winning choices instead of one, you would double his/her chances of winning (approximately). Actually, since a student would not choose the same number with both choices, the chances of winning would be slightly greater than double (non-replacement). A student's first selection would have a 1 in 20 chance of winning and his/her second selection would have a 1 in 19 chance of winning.

b) Reducing the number of wrong choices by decreasing the number of combinations possible. Anything that reduces the number of total combinations from which the students choose would increase the probability of the winning combination being selected. This could be accomplished by -

* Having a smaller pool of integers from which to choose. Instead of having three numbers out of six, you could have three numbers out of five, for example. Total number of combinations would then be reduced from 20 (6x5x4/3x2x1) to 10 (5x4x3/3x2x1). Another way of reaching this result (smaller number pool) would be to take out all even numbers, for example. Since there are only three even numbers among the first six integers and students get to choose three numbers, this would increase everyone's chance of winning to one-in-one.

* Increasing the number of integers in the winning combination without increasing the number of possible integers to choose from. For example, the winning combination could be 4 integers chosen from 6 integers. This would reduce the total number of combinations from 20 to 15 (6x5x4x3/4x3x2x1).

* Decreasing the number of integers in the winning combination without changing the number of integers in the selection pool. For example, having the students choose only two numbers instead of three would reduce the number of combinations from 20 to 15 (6x5/2x1).

c) Screening the student choices to prevent duplication and giving those with duplicate choices a chance to select another number. This would not increase the chances of winning for any individual student, but it would increase the chances of some student winning because more different combinations would be selected.

To the extent that duplicates are eliminated, this would increase the number of combinations that the students as a group could select and have the same effect (for the group) as answer “a” above.

d) Drawing two (or more) potential winning combinations each time instead of one. This would increase everyone’s chances from one-in-twenty to two-in-twenty (if you put the ping-pong balls back into the bin before choosing the second combination). The chances of choosing a winner would increase even more if the balls were not replaced, and, in fact, you could ensure a winner merely by continuing to draw until a winning combination is drawn.

e) Other - Students may come up with other valid answers.

2. How could you alter the rules and/or structure of our class lottery game in order to decrease the chances of anyone winning? List as many different ways as you can and explain your reasoning mathematically.

SAMPLE ANSWER - You can decrease the chances of winning either by decreasing the number of choices or by increasing the total number of combinations and thereby the total number of wrong choices.

a) Since each student now has only one choice, it would be impossible to reduce the number of choices, but it would be possible to condition the choice on another event with a low probability of occurring. For example, if a student had to roll a six on a six-sided die in order to choose a combination in the lottery, that would reduce his/her chances of winning by 5/6 since the chance to participate in the lottery would be one in six.

b) You could also increase the total number of integers in the pool so that the total number of combinations would be increased. For example, if the students are asked to choose three numbers out of nine, then the chances of choosing a winning combination would go down from one-in-twenty to one-in-eighty-four (9x8x7/3x2x1).

c) You could also change the requirement so that the winning set of three numbers would have to be in the same order as the numbers drawn on the ping pong balls. this would change the possibilities from combinations to permutations and increase the number of possibilities from 20 to 120 (6x5x4, not divided by 3x2). In this case, for example, 4-3-1 would not be a winning selection if the numbers drawn on the ping pong balls were 4, 1 and 3 in that order.

d) You could instruct the students to choose any three-digit number using the first six integers and replace the ping-pong balls after each number is drawn so that the numbers containing duplicate integers are added to the pool of possible combinations (or permutations, if you also decide to make that change simultaneously).

This would decrease the chances of winning to 1-in-216 in the case of permutations (three independent events, each with a one-in-six probability) or 1-in-? in the case of combinations.

e) Other - Students may come up with other valid answers.

Lottery Assessment

You and your classmates enjoyed the class lottery so much that you decided to run a school wide lottery to demonstrate the probability of winning in the state LOTTO games. However, most of the student body do not know about the probability of winning a lottery. So you decide to create two different lotteries -- one that is easier to win and one that is harder to win. You design a lottery with a high probability of winning to use in each classroom that has a prize of extra credit points. Then you create a lottery for the student body that has a low probability of winning that gives a "Get Out of Class Free" card as a prize.

Think about what the probability of winning in the classroom lottery should be. Think about how many numbers should be picked and what the total amount of numbers should be. Now think about how you would set it up for the school wide lottery.

Now create two different lottery games that could be used. Be sure to explain clearly how many numbers should be picked and how many total numbers will be used. Be sure to justify your games by including mathematical proof of your probabilities of winning.

Lottery Assessment Rubric

Response Levels:

3 Accomplishes the creation of an alternative game
Demonstrates an understanding of lottery probability
Shows correct mathematical proof

2 Partially creates an alternative game
Demonstrates a partial understanding of lottery probability
Mathematical proof has errors

1 Partially creates an alternative game
Demonstrates a fragmented understanding of lottery probability
Mathematical proof is incorrect or missing

Lottery Assessment Hints for Written Assessment

1. Use MSPAP paragraph form (clear topic sentence, at least three details of math content, concluding sentence) to answer prompt. Math details should justify the probabilities presented.

2. Be sure to organize in a logical format. (Put high probability game proof together and not mixed in with low probability game proof .)

3. May be either typed or neatly hand written. (Should have a cover page.)

Lottery Assessment Hints for Oral Assessment

1. Should have an appropriate and clear visual aid.

2. Should use index card notes. (Do not simply read to the class.)

3. Should be between 3 to 5 minutes in length.

GRADING RUBRIC for ______________________
peer grading
(good - 3, fair - 2, poor - 1)

High probability game: __________ clear explanation of game rules

__________ clear mathematical explanation

Low probability game: __________ clear explanation of game rules

__________ clear mathematical explanation

GRADING RUBRIC for _____________________
peer grading
(good - 3, fair - 2, poor - 1)

High probability game: __________ clear explanation of game rules

__________ clear mathematical explanation

Low probability game: __________ clear explanation of game rules

__________ clear mathematical explanation

GRADING RUBRIC for ___________________
peer grading
(good - 3, fair - 2, poor - 1)

High probability game: __________ clear explanation of game rules

__________ clear mathematical explanation

Low probability game: __________ clear explanation of game rules

__________ clear mathematical explanation

GRADING RUBRIC for __________________
peer grading
(good - 3, fair - 2, poor - 1)

High probability game: __________ clear explanation of game rules

__________ clear mathematical explanation

Low probability game: __________ clear explanation of game rules

__________ clear mathematical explanation